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ECONOMIC CYCLES AND TERM STRUCTURE: APPLICATION TO BRAZIL

**PRISCILA FERNANDES RIBEIRO
PEDRO L. VALLS PEREIRA**



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Economic Cycles and Term Structure: Application to Brazil

Priscila Fernandes Ribeiro
ITAU-UNIBANCO
Av Engenheiro Armando de Arruda Pereira n 707
Torre Conceição - 4 andar
04344-902, São Paulo, S.P.
BRAZIL
e-mail: priscila.ribeiro@itau-unibanco.com.br

Pedro L. Valls Pereira*
CEQEF and EESP-FGV
Rua Itapeva 474 sala 1202
01332-000, São Paulo, S.P.
BRAZIL
Phone: +55+11+3799-3726
Fax: +55+11+3379-3350
e-mail: pedro.valls@fgv.br

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Abstract

The objective of this work is to describe the behavior of the economic cycle in Brazil through Markov processes which can jointly model the slope factor of the yield curve, obtained by the estimation of the Nelson-Siegel Dynamic Model by the Kalman filter and a proxy variable for economic performance, providing some forecasting measure for economic cycles.

Key Words: Dynamic Nelson & Siegel; Term Structure of Interest Rate; Business Cycles; Kalman Filter; Markovian Switching.

JEL Code: G12; E37

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1 Introduction

There is evidence that the yield curve shows cyclical behavior, correlated with future economic expansions and recessions¹. In general, the yield curve has a positive slope. This is the case observed during the initial periods of economic expansions, when economic agents expect an increase in short-term interest rates. By the arbitrage and liquidity preference theories, in accordance with Campbell, Lo & MacKinlay (1997), in order to acquire long-term securities instead of securities with risk-free short-term maturities, investors demand a risk premium.

On the other hand, the slope of the yield curve tends to become flat or inverted at the end of expansion periods (start of the recession). A possible explanation is the presence during these periods of restrictive monetary policy. By another explanation, according to the theory of expectations, long-term rates reflect expectations of agents on the future of short-term rates, so that a flat yield curve would indicate that the market is expecting a fall in real future interest rates, given the probability of weaker economic performance in the future.

There is a vast literature in forecasting of economic activity and discrete choice models using the term structure of interest rates². For this literature, linear regression models are used to forecast the rate of economic growth, and the discrete choice models (Probit and Logit) used to forecast the probability of economic recession, basically using the slope of the curve.

Ang, Piazzesi & Wei (2006) nevertheless show that the use of all of the information in the yield curve may result in more precise estimates of real economic growth. Furthermore, the non-linearities imposed during the transition phases from one regime to another may capture structural changes which cannot be perceived by linear models.

The Markov process for the estimated slope factor for the yield curve represents the cycles in the securities market, showing a relationship with economic cycles, for which reason, it is used as a lead indicator. For the purposes of comparison with the results of the model, we have used the dates of economic cycles within Brazil provided by CODACE.

2 Economic Cycles, the Yield Curve and CODACE

During the 1990s, Estrella & Hardouvelis (1991) were the first to test the spread empirically as a predictor of economic cycles. The work showed that a positive slope for the spread of interest rates implied higher GDP growth and that an increase in the spread implies a reduction in the probability of a recession four quarters in the future.

¹See, for example, Harvey (1988, 1989); Stock & Watson (1989); Ang, Piazzesi & Wei (2006), and for a review of the literature, Stock & Watson (2003).

²Harvey (1988, 1989), Stock & Watson (1989), Ang, Piazzesi & Wei (2006).

Estrella & Mishkin (1998) compare the performance of various financial variables, including the spread and the equity index, among other lead indicators, and show that the term structure of interest rates has strong predictive power compared with the other indicators tested. According to Moolman (2004), the relationship between the economic cycle and the term structure of interest rates is such that when the economy is in an accelerated growth phase, there is a general consensus between investors that the economy is moving towards a deceleration or recession in the future. In this way, investors may seek to protect themselves against recession by acquiring financial assets (e.g. by acquiring long-term securities), which will produce returns during the economic contraction. The increase in demand for long-term securities will cause an increase in their price, implying a reduction in the return on long-term securities. In order to finance these purchases, investors sell their short-term assets, causing a decline in these prices, and consequently, an increase in the yields of these assets. In other words, if a recession is expected, in the long term, interest rates will fall and short-term interest rates will rise.

Consequently, prior to a recession, the slope of the term structure will become inverted. Within Brazil, Tabak & Andrade (2001) use the expectations hypothesis to analyze the term structure with daily data. Lima & Issler (2002) tested the rational expectations hypothesis for monthly data. These studies concluded, albeit only partially, in favour of this hypothesis. Marçal & Valls Pereira (2007), using cointegration techniques, nevertheless found evidence against this hypothesis.

The Economic Cycles Dating Committee, CODACE, of the Getúlio Vargas Foundation, establishes a reference chronology for economic cycles. The methodology is developed on the basis of the following facts: each local maximum point (peak point) of the cycle is equivalent to end of an expansion period, which will be followed, in the following quarter, by the start of a recession; each local minimum point (trough point) is equivalent to the final quarter of a recession, to be followed, in the following quarter, by the start of an economic expansion. The points of transition, according to the CODACE report of May 2009, were determined by the Committee pursuant to classic concepts of expansion and recession adapted to the “peculiarities of the Brazilian economy”.

Figure 1, constructed on the basis of a monthly GDP series published by the Central Bank and which we shall discuss in more detail later in the article, presents the dates drawn up by CODACE for the period including the first quarter of 2000 until the third quarter of 2009. According to the estimates of the Committee, between 1980 and 2008, the average duration of recessions was approximately 6 months and of expansions 11 months.

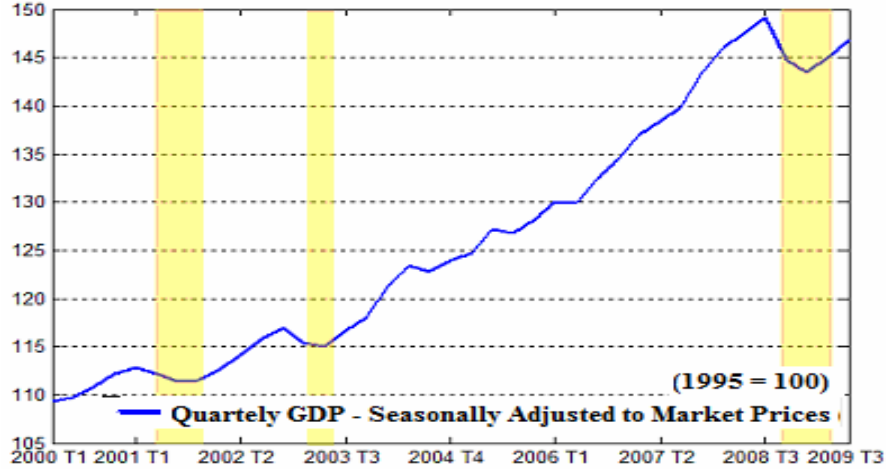


Figure 1: Quarterly Timeline of Brazilian Business Cycles.

In Figure 1, the shaded areas indicate periods characterized by recession. The dating of CODACE, considered recessions to be periods in which there was a sharp decline in the level of economic activity perceived for at least two consecutive quarters. The principal variable used in the dating of CODACE was the seasonally-adjusted quarterly gross domestic product (GDP) at market prices, calculated by the IBGE. Periods considered as recessions are respectively, from the second to the fourth quarter of 2001; from the first to the second quarter of 2003; and from the fourth quarter of 2008 onwards. According to the IBGE, the accumulated growth rates for these periods were -1% , -1.7% and -3.6% respectively (the last figure in the fourth quarter of 2008 alone). The first period considered as a recession refers to the second and fourth quarters of 2001. During this period, GDP fell for three consecutive quarters, caused principally by the energy crisis in Brazil, the attacks on the World Trade Center and the effects of the bursting of the technology bubble in the United States. The recovery at the end of the year caused annual GDP to end with an increase of 1.3% . The second period considered as a recession refers to the first and second quarters of 2003. During this period, the Brazilian economy contracted by 1.44% during the first quarter and by 0.23% in the second. In that year, however, GDP recovered and closed with an increase of 1.1% . The causes were fundamentally the appreciation in the dollar which occurred in the preceding months due to the negative effects of the sharp deceleration in the growth of the world economy on the renewal of credit lines for emerging countries and, in particular, of the expectations of economic agents regarding the management of the public sector debt by the Lula government, which took power in January 2003.

The third period considered as a recession by CODACE refers to the period between the fourth quarter of 2008 and the first quarter of 2009, characterized by a sharp overall fall in real Brazilian GDP of 3.8% , according to a report pub-

lished on 28 December 2009, which was thus the largest average reduction since 1980. For the Committee, the resumption of expansion by part of industry, the sector most affected by the crisis, is an indicator of the end of the recession. During this period, the crisis, which originated in the United States and was initiated by the US property market, caused instability in the financial markets and in the exchange rate, together with a declining trend in equity prices, [with these] representing elements which impacted the real economy, as they reduced the volume of productive investments. In its last report of 28 December 2009, CODACE stated, by using the GDP, production, sales, employment and income data that the period of recession in Brazil had ended, given that there was a trough in the Brazilian economic cycle during the first quarter of 2009. According to the Committee report, the trough represents the end of a recessionary period and the start of a period of economic expansion.

3 The Dynamic Nelson-Siegel Model and MS-VAR Representation of a three-factor model

The approach in Nelson-Siegel (1987) describes the yield curve through exponential weighting of the factors. In this approach, the proposed model is capable of representing a large number of maturities for interest rates as a mathematical function. The authors argue that these functions may be used to obtain a parsimonious model, representing the principal stylized facts, historically observed in the yield curve: monotonicity, convexity and a ‘bell’ shape, persistence of the level of interest rates, higher volatilities for short-term rates and low persistence of *spreads*³.

The Nelson-Siegel representation of the yield curve is modified in Diebold & Li (2006). The authors show that the above representation may be interpreted as a model of latent factors and β_{1t} , β_{2t} and β_{3t} are the level, slope and curvature, which vary over time, being weighted by a fixed component λ (*factor loadings*). A change in the long-term factor, β_{1t} , reproduces the same change in all of the interest rates, regardless of their maturity. Diebold & Li (2006) identify this component for the US yield curve with a high degree of persistence, being associated with the inflation expectations of economic agents. In analogous fashion, an increase in the short-term factor, β_{2t} , shall have an effect more predominantly on the short-term interest rates, affecting the slope of the curve. Diebold & Li (2006) associate this factor with the expected behaviour of the economy, such as the expected level of economic activity. In this way, this component is directly related to economic cycles. An increase in the medium-term factor, β_{3t} , will not immediately affect long-term interest rates, but will have a greater effect on medium-term rates, indicating that the curvature of the yield curve will vary over time, reproducing the stylised fact of a ‘bell’ form of the yield curve.

It is also worth commenting that the factors β_{1t} , β_{2t} and β_{3t} recovered

³The differential of the long-term interest rate to the short term interest rate.

by Diebold & Li (2006) have a high degree of correlation with the empirical measures of level, slope and curvature which are commonly used⁴. This result is highly desirable insofar as the methodology of Nelson & Siegel could not be considered appropriate if the factors deriving from it (which depend on the pre-specified functional forms for the decay parameters) did not resemble the factors deriving from what economic agents understand by measures of level, slope and curvature, as set forth in Diebold, Rudebusch & Arouba (2006). In this way, the interest rates, observed as a time series, may be described jointly for the different maturities by the following regression model:

$$y_t(\tau) = \mu_\tau + \psi_{1\tau}\beta_{1t} + \psi_{2\tau}\beta_{2t} + \psi_{3\tau}\beta_{3t} + \xi_{t\tau} \quad (1)$$

for $t = 1, \dots, T$. $\xi_{t\tau} \sim N(0, \Sigma_\xi)$ where Σ_ξ is a diagonal matrix. The preceding assumption implies that changes in interest rates of different maturities are not correlated.

In this regression model, μ_τ is a constant for each maturity of interest rates and may be interpreted as a fixed effect insofar as each constant reflects the particular characteristics for each maturity of interest rates which are observable and non-observable. In addition, despite the fact that within Brazil, the yield curve suffered a multitude of shocks during the selected period, altering its behaviour in terms of level and volatility, there is no loss of generality in assuming that the constant for every maturity, μ_τ , does not vary over time, in addition to which, we avoid dictating a behavioural rule for the parameter, as well as the number of parameters to be estimated; β_{1t} , β_{2t} and β_{3t} are the factors which vary over time and $\psi_{j\tau}$ is the weight for the factor j and maturity τ .

As made explicit in Diebold & Li (2006), the parameters $\psi_{j\tau}$ may be considered constant over time. In the work of Koopman, Mallee & Van der Wel (2008), using the monthly data of US Treasury *zero coupon bonds*, provided by Fama-Bliss, they argue that maintaining the decay factor λ fixed during the study period may be highly restrictive, since the slope factor β_{2t} and the curvature factor β_{3t} are merely dependent on λ . Diebold & Li (2006) argued that the gain for the adjustment and predictive power of the model is small when the decay parameter is allowed to vary.

If β_{it} is a vector of latent variables, it may be shown that its representation may be made through vector autoregression (VAR) proposed in Diebold & Li (2006). In this case, the dynamic of β_{1t} , β_{2t} and β_{3t} may be described by a VAR(1) model.

Pursuant to the account and according to Diebold & Li (2006), this structure is already in a state space representation. According to Koopman, in an empirical study of the US yield curve, whose estimation methodology for the yield curve shall be adopted in this study, the factors which describe the same

⁴We define the level as being the mean of all of the yields; slope as being the difference between the long-term and short-term yields; and curvature as being equal to twice the medium-term yield subtracted from the sum of the short and long-term yields

have a long memory. Due to this fact, the process proposed by the authors of the factors is a random walk:

$$\begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} = \begin{pmatrix} \beta_{1t-1} \\ \beta_{2t-1} \\ \beta_{3t-1} \end{pmatrix} + \begin{pmatrix} \zeta_{1t} \\ \zeta_{2t} \\ \zeta_{3t} \end{pmatrix} \quad (2)$$

for $t = 1, \dots, T$. The initial conditions are $\beta_1 \sim N(0, \Sigma_\beta)$, Σ_β diagonal, and $\zeta_1 \sim N(0, \Sigma_\zeta)$, Σ_ζ is not necessarily diagonal, i.e. the factor errors may be simultaneously correlated. This process, proposed in Koopman (2007), is a particular case of the process proposed in Diebold & Li (2006). In this way, we may test whether the proposed $VAR(1)$ may be reduced to a random walk for the latent factors.

In order to estimate the yield curve described above, the approach used shall be the Kalman filter, as used by Diebold, Rudebusch & Arouba (2006) and described in Koopman (2007). The authors implemented the simultaneous estimation of the observation and transition equations, obtained the factor weights for each maturity and the dynamic between the factors which describe the interest rates. Diebold, Rudebusch & Arouba (2006) explain that single stage estimation is better because it is able to consider all of the uncertainties associated with the estimation of these parameters in a single step. De Pooter (2007), comparing various classes of the Nelson-Siegel model with monthly data for US Treasury zero coupon bonds of Fama-Bliss for the period from January 1984 to December 2003, found that the three-factor model with a decay parameter, λ , appearing and estimated in a single stage by the Kalman filter, has a mean square error less than the same model estimated by two-stage least squares.

The Kalman filter is an algorithm used for the linear prediction of the state vector, in this case, the latent variable vector, conditional on the observed variables, interest rates. Under normality, the likelihood function of the model is obtained through the prediction error decomposition. Once the likelihood function has been obtained, the coefficients are estimated by numerical methods. After the estimation of the parameters through the smoothing algorithm, it is possible to recover the smoothed state vector and to obtain some structural interpretation for these estimates. The likelihood function of the system depends on the prediction errors, their respective variances and the set of parameters, $\Theta = (\mu_\tau, \lambda, \beta_{it}, \Sigma_\zeta, \Sigma_\xi)$ and is given by:

$$L(\Theta) = \sum_{t=1}^T \left[-\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(|F_{t-1}|) - \frac{1}{2} \varsigma_{t|t-1}^\top F_{t-1}^{-1} \varsigma_{t|t-1} \right] \quad (3)$$

where $\varsigma_{t|t-1} = y_t(\tau) - y_{t|t-1}(\tau)$ is the one-step forward prediction error and $F_{t-1} = E(\varsigma_{t|t-1} \varsigma_{t|t-1}^\top)$ is the variance of the prediction error.

4 Vector Autoregressive with Markovian Switching (MS-VAR)

The $MS - VAR$ models arise from the union of two tools: VAR , introduced by Sims (1980) and models which use Markov Switching to analyse the nature of regime changes in macroeconomic series, as developed by Hamilton (1989) on economic cycles in the United States. With this, it becomes possible to estimate VAR models subject to regime changes.

In the analysis of time series, the introduction of the Markov Switching model is due to Hamilton (1988) and Hamilton (1989), with this latter study having inspired more recent contributions. The class of $MS - VAR$ models provides a convenient structure for analysing multivariate representations with regime changes. These models permit various dynamic structures, depending on the value of the state variable, s_t , which controls the transition mechanism between the various states. The $MS - VAR$ model belongs to a more general class of nonlinear models that, conditional on a particular regime, the model is linear.

In this way, the $MS - VAR$ model may be described, pursuant to Krolzig (2003), as an autoregressive process of observed time series $\mathbf{y}_t = (y_{1t}, y_{2t}, \dots, y_{kt})$ the parameters of which are unconditionally variant over time, but constant when conditional on a discrete non-observed state (or regime) variable $s_t \in \{1, 2, \dots, m\}$:

$$\mathbf{y}_t - \boldsymbol{\mu}(s_t) = A_1(s_t)[\mathbf{y}_{t-1} - \boldsymbol{\mu}(s_{t-1})] + \dots + A_p(s_t)[\mathbf{y}_{t-p} - \boldsymbol{\mu}(s_{t-p})] + B(s_t)\mathbf{u}_t \quad (4)$$

$$\text{where } \boldsymbol{\mu}(s_t) = \begin{cases} \boldsymbol{\mu}_1 = (\mu_{11}, \dots, \mu_{\tau 1}) & \text{if } s_t = 1 \\ \vdots & \\ \boldsymbol{\mu}_m = (\mu_{1m}, \dots, \mu_{\tau m}) & \text{if } s_t = m \end{cases}, \mathbf{u}_t \text{ is a Gaussian error}$$

conditional on regime s_t : $\mathbf{u}_t | s_t \sim NID(0, \Sigma(s_t))$. p is the order of the VAR , m is the number of unobserved regimes and k is the dimensions of the vector of observed variables. Hence, this model may be described as being of type $MS(m) - VAR(p)$. The transition functions in the matrices of parameters $\boldsymbol{\mu}(s_t)$, $A_1(s_t)$, \dots , $A_p(s_t)$ and $\Sigma(s_t)$ describe the dependence of the VAR parameters in each regime. The important characteristic of a model with Markovian change is that the unobserved realisations of the regime $s_t \in \{1, 2, \dots, m\}$ are generated by a discrete time stochastic process which may be represented by a Markov chain with finite states⁵, defined by its transition probabilities:

$$p_{jl} = \Pr(s_{t+1} = l \mid s_t = j) \quad \sum_{l=1}^m p_{jl} = 1 \quad \forall j \in \{1, 2, \dots, m\} \quad (5)$$

⁵The difference between the Markovian models and the models with a threshold autoregressive - TAR, STAR and SETAR is that in these models, the variable which determines the regimes is observed.

in which it is accepted that the Markov chain is irreducible and ergodic⁶.

In the model described by the equation (4) there is an immediate jump in the mean of the process after a regime change. It is frequently more probable to assume that the mean is modified smoothly to a new level after transition from one regime to the other. In this situation, we may use a model with an intercept term, $\mathbf{v}(s_t)$, depending on the regime. We thus have:

$$\mathbf{y}_t = \mathbf{v}(s_t) + A_1(s_t)\mathbf{y}_{t-1} + \dots + A_p(s_t)\mathbf{y}_{t-p} + B(s_t)\mathbf{u}_t \quad (6)$$

Constraining the linear *VAR* models and parameters which are invariant over time, the mean form, adjusted by (4) and with the form of an intercept in (6) are not equivalent. These entail adjustment dynamics for the observed data, which are different after a regime change. While a permanent change in the regime by the mean $\boldsymbol{\mu}(s_t)$ causes an immediate jump in the time series vector observed for a new level, the dynamic response for a regime change in the intercept term $\mathbf{v}(s_t)$ is equivalent to a “shock” in the white noise series \mathbf{u}_t .⁷

The inference with a view to dating regimes not observable in *MS – VAR* is made basically on the basis of the filtering and smoothing of estimated probabilities.

The filtering method is usually the algorithm of Hamilton (1989), but other filters may be used, such as the Kalman filter. Filtering permits inferences on the probability distribution of the non-observed regime s_t , given the information set \mathbf{y}_t .

The *EM* algorithm is an approach broadly applied to the estimation by maximum likelihood. The estimation procedure for the parameters is the maximisation of the likelihood function and its use for inferences on non-observable states, s_t . This method nevertheless becomes less attractive as the number of parameters grows. In this case, the *EM*, originally described by Dempster, Laird & Rubin (1977) to recover missing observations from a normally distributed sample is more appropriate. This methodology starts with estimates of omitted observations, and through iterative procedures, results in a new joint distribution which increases the probability of occurrence of a given observation. The adaptation of this procedure to time series data was described by Schumway & Stoffer (1982), also in a context of missing observations.

According to Hamilton (1989), the algorithm consists of two parts. In the first part, population parameters, including the joint probability density of the states, are estimated, and in the second part, probabilistic inferences are made on the unobserved states, using a non-linear filter and smoother.

In this way, let s_t be a given regime and $\mathbf{Y}_{t-1} = (y_{t-1}, \dots, y_{t-p})$ the endogenous variables used in the analysis. Considering that the error \mathbf{u}_t has a normal

⁶There are various definitions of the term “ergodic”. For some authors, the finite property is that the initial state may be forgotten, for others that the mean converges over time independently of the initial state. In general, these definitions are not equivalent. Some definitions exclude chains with transitory states having zero equilibrium probabilities. For further details, see Hayashi, 2000.

⁷For further details see: Hamilton (1989).

distribution and that the process is in the regime $s_t = i_l$ in t , the conditional density of \mathbf{y}_t is given by:

$$f(\mathbf{y}_t \mid s_t = i_l, \mathbf{Y}_{t-1}; \omega) = \ln(2\pi)^{-1/2} \ln |\Sigma|^{-1/2} \exp \left\{ (\mathbf{y}_t - \bar{\mathbf{y}}_{i_l})^\top \Sigma^{-1} (\mathbf{y}_t - \bar{\mathbf{y}}_{i_l}) \right\}$$

where i_l represents the l -th column of the identity matrix \mathbf{I}_N , $\bar{\mathbf{y}}_{i_l} = E(\mathbf{y}_t \mid s_t = i_l, \mathbf{Y}_{t-1})$ is the conditional expectation of \mathbf{y}_t given that the process is in i_l and ω is a vector which contains the population parameters, including the autoregression parameters Θ , and the transition probabilities governing the Markov chain of the unobserved states.

The information on the realisation of Markov states are collected in the vector s_t , which consists of binary variables defined on the basis of a function indicating each regime.

The conditional densities, for the m possible regimes, are given below:

$$\gamma_l = \begin{bmatrix} f(\mathbf{y}_t \mid s_t = i_1, \mathbf{Y}_{t-1}) \\ \vdots \\ f(\mathbf{y}_t \mid s_t = i_m, \mathbf{Y}_{t-1}) \end{bmatrix}$$

In order to obtain the marginal density of \mathbf{y}_t two steps are followed, as set forth above. In the first, the joint density of \mathbf{y}_t and s_t is written as a product of the marginal and conditional densities. In the second, it is integrated with regard to all of the regimes. The results of the marginal density of \mathbf{y}_t may be interpreted as a weighted average of the conditional densities, where the weights are the probabilities of the regimes. In this way, we must make some inference on the unobserved regimes, via the BLHK (Baum-Lindgren-Hamilton-Kim) filter and smoother, used in this work, which permits inferences on the process states through filtered and smoothed probabilities.

The final results of the *EM* algorithm shows that the value of the likelihood function increases with the number of iterations. At a determined fixed point for these iterations, the estimated parameters converge to the maximum likelihood estimators. In many studies using a regime change⁸, the strategy adopted to determine the optimal number of regimes to be introduced into the model is based on economic theory and on stylised economic facts. Having chosen the optimal number of regimes, the standard tests of *VAR* models (information criteria, likelihood ratio test or Wald test) may be used to choose the best model. There is a difficulty in implementing tests for the choice of number of regimes: under the null hypothesis, some parameters cannot be identified. In this way, the likelihood ratio tests (LR) do not have a standard asymptotic distribution and cannot be adopted⁹.

Based on Hamilton (1989), we initially consider a model in the state space representation restricted to two regimes, where $s_t = 1$ indicates a recession regime and $s_t = 0$ an expansion regime.

⁸ Silva, Silva & Frascaroli (2008); Gadea, Loscos & Montañés (2006).

⁹ Hansen (1992) & García (1993).

As discussed above, an economy may be considered to be in recession when its GDP falls for at least two consecutive quarters and vice versa. It should be noted that for the period analysed here, the state observe for the Brazilian economy predominantly corresponds to an expansion regime, even though it has shown a pattern which is informally termed “*stop-go*”. In other words, the Brazilian economy has alternated between low growth, or recession, and phases of accelerated growth, suggesting that we could adopt more than one form of classifying regimes. At the same time, within Brazil, there is no official disclosure by the Central Bank of business cycles within Brazil and only recently and unofficially by CODACE, and it is on the basis of these datings of CODACE that we shall evaluate the performance of our model.

In this way, the estimated model presents the following form, in which the first equation is of observation and the second of transition, so that for the representation of the state space we have:

$$\begin{aligned} \mathbf{y}_t &= \Phi_{s_t}^1 + \Phi_{s_t}^2 \mathbf{y}_{t-1} + \mathbf{\Lambda}_{s_t} \mathbf{F}_t + \mathbf{U}_t \\ \mathbf{F}_t &= \boldsymbol{\alpha}_{s_t} + \Psi \mathbf{F}_{t-1} + \mathbf{N}_t \end{aligned} \quad (7)$$

where now $\mathbf{y}_t = \{y_t(3m), y_t(12m), y_t(60m), \Delta_{12}g_t\}$, $y_t(\tau)$ are the interest rates for 3,12 and 60 months and $\Delta_{12}g_t$ the 12-month change in GDP; $\mathbf{F}_t = \{\hat{\beta}_{2t}, \mu_{g_t}\}$ and $\hat{\beta}_{2t}$ are the estimated latent factors for the slope and μ_{g_t} is the components of the trend in 12-month GDP growth rate; $\mathbf{\Lambda}$ is a 4×2 matrix of weights, \mathbf{U}_t , a vector of random components, which measure the movements not captured by the model; $\mathbf{\Lambda}_{s_t}$ a vector of weights which portrays the effects of factors on the vector of interest rates and change in the GDP. By hypothesis, the factors are not correlated with the errors \mathbf{U}_t for all of the lags. $\mathbf{N}_t \sim N(0, \Omega_t)$ and Ω_t is a diagonal variance and covariance matrix for the factors.

Each factor follows an unobserved autoregressive process in which all the parameters are functions of two distinct Markov processes, $s_t^{\hat{\beta}_{2t}}$ for the yield curve factor and $s_t^{\mu_{g_t}}$ for the economic factor. The Markov chain $s_t^{\hat{\beta}_{2t}}$ represents the rising phase ($s_t^{\hat{\beta}_{2t}} = 1$) and the falling phase ($s_t^{\hat{\beta}_{2t}} = 0$) in the securities market, since the chain $s_t^{\mu_{g_t}}$ represents expansion ($s_t^{\mu_{g_t}} = 1$) and recessions or stagnation ($s_t^{\mu_{g_t}} = 0$) of the economic cycle. Given the hypotheses of the model, the representation allows the process for the cycle in the securities market and for the economic cycle to be independent or unsynchronised over time.

5 Estimations and Results

5.1 Database

The data used in this study are the closing monthly prices for DI-PRÉ swap rates. The maturities used were 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 48 and 60 months. The data were observed for the period from March 2000 to

August 2009. Some descriptive statistics for the DI-Pré swap data are presented in Table 4, in the annex. Figure 7 shows the mean and median yield curve for the analysed period. The mean yield curve shows the normally observed behaviour, i.e. an increasing trend at maturity. The median yield curve shows flat behaviour (without a slope), an indication that during the analysed period, it is expected that the short-term rates will continue the same. I.e. it is expected that with regard to interest rates, economic policy will remain the same.

The yield curve for the study period considers various formats, with various changes in slope and curvature, assuming rising and inverted formats on various occasions during this period. Figure 2 shows a three-dimensional graph of the studied curve.

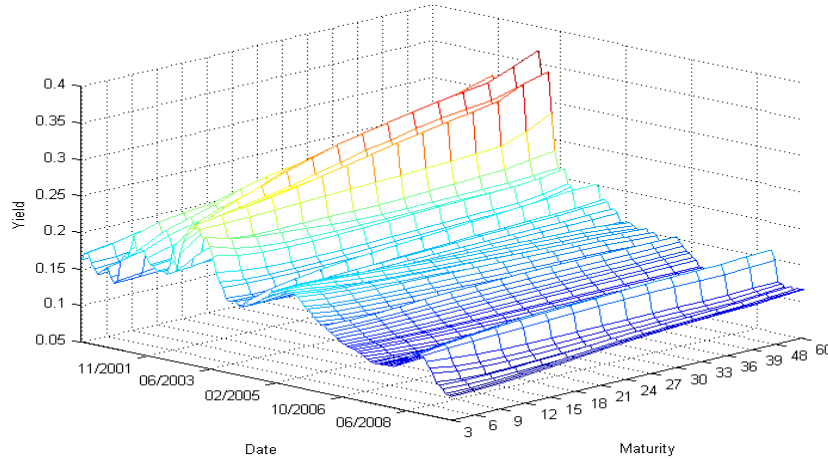


Figure 2: Brazilian Yield Curve - March 2000 to August 2009.

The analysis of the graph of Brazilian GDP shows a constant growth trend with falling periods, indicating a fall in economic activity during the period of analysis of this study. This behaviour may be seen with a monthly frequency, presented in Figure 1. The data refer to the series for Brazilian GDP during the period March 2000 to August 2009, calculated by the IBGE and published by the Brazilian Central Bank (BCB) (Series 4380).

However, according to the work of Chauvet and Senyuz (2009), we shall use the 12-month growth rate for GDP, set forth in Figure 3. For this series, the ADL and KPSS tests were carried out. In the ADL test, we reject the null hypothesis with a test statistic of -3.885509 and a critical value of -2.8903 at the 5% significance level, indicating that this series is stationary. For the KPSS test, we do not reject the null hypothesis of stationarity, with a test statistic of 0.6054 in the face of a critical value of 0.7390 at the 5% significance level.

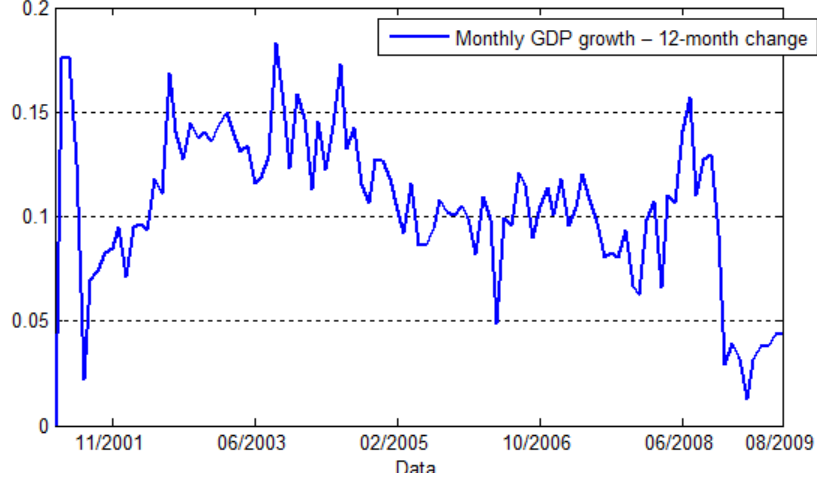


Figure 3: Monthly GDP growth – 12 month change

5.2 Results

In this section, the principal results are presented on business cycles in Brazil, using the $MSIAH(2) - VAR(1)$ class of models, for the real GDP 12-month change data and the slope factor estimated by Kalman filter, for the period from March 2000 to August 2009.

The parameters $\psi_{j\tau}$ and the factors β_{jt} were estimated by Kalman filter. Estimates were also obtained for the variance and covariance matrices Σ_ξ and Σ_ζ . For the identification of all of the parameters $\psi_{j\tau}$, the weights (*factor loadings*) of the three factors for the maturities of 3, 12 and 60 were restricted to $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, respectively. In this way, the maturity of 3 months is directly related to the slope factor. The maturities of 12 and 60 months were linked to the curvature and level factors respectively.

The weights estimated by Kalman filter are presented in Figure 4.

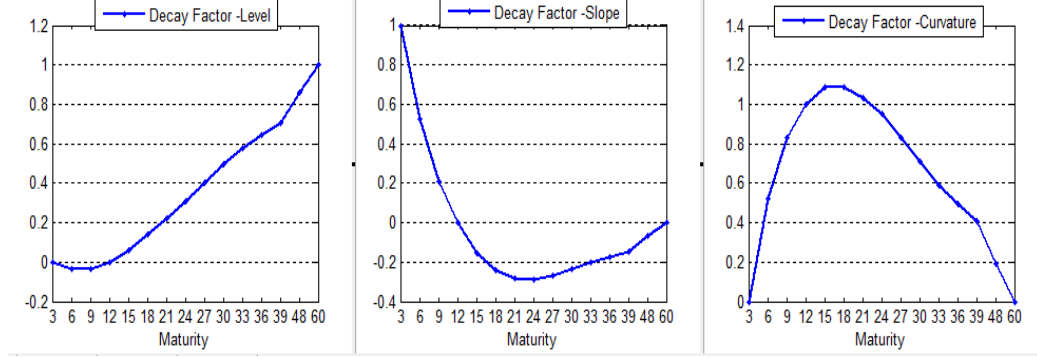


Figure 4: The weights estimated by Kalman filter

As mentioned, the weights for the level factor are associated with longer maturities of 48 and 60 months. The second and third factors are associated with the short and medium term maturities respectively. Observing the above figures, the estimates for the weights of the second and third factors, slope and curvature, for the proposed model are very similar to the restricted weights proposed in Nelson-Siegel (1987).

The smoothed estimates of the factors are presented in Figure 5.

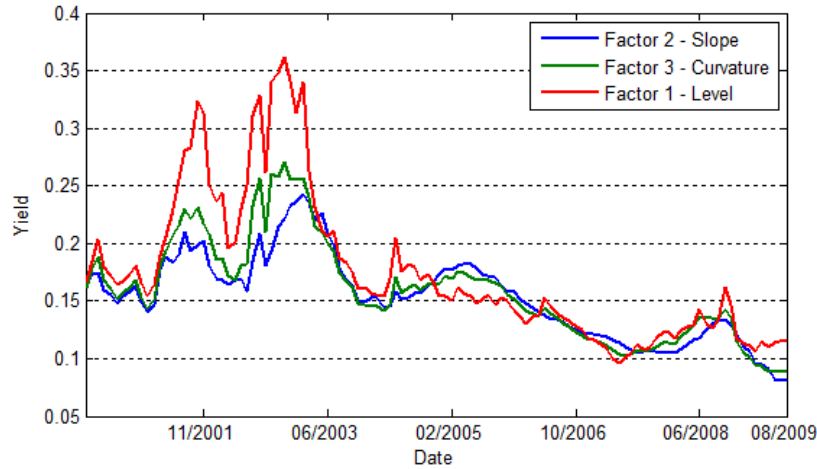


Figure 5: Interest Rates - 3, 12 and 60 months

The analysis of the parameters of the $VAR(1)$ model indicates a high degree of persistence in the dynamic of the three latent factors and there are apparently significant cross-effects for the dynamic of the factors. Factor 3 influences the factors 1 and 2. The results are presented in Table 1.

	β_{1t}	β_{2t}	β_{3t}
β_{1t-1}	0.59061 (0.09775)	-0.11774 (0.14396)	-0.47801 (0.23575)
β_{2t-1}	0.520483 (0.14467)	1.144207 (0.21305)	0.780721 (0.34891)
β_{3t-1}	-0.118191 (0.05260)	-0.033259 (0.07747)	0.705295 (0.12686)

Table 1: Estimated Parameters of the $VAR(1)$ for the Smoothed Latent Factors (Standard Error in parentheses)

The autovalues of the matrix of coefficients of $VAR(1)$ above are 0.5493, 0.9935 and 0.8973. Since one of the auto values is close to unity, the components of VAR are as a minimum integrated of order 1, and like the other two autovalues, are less than unity, with this indicating, together with the prior statement, that the components are at most integrated of order 1.

We propose a two-factor model, which takes into consideration the dynamic between the securities market and business cycles. This model uses the components of the yield curve to determine economic cycles with greater precision, analysing this through the 12-month growth rate of GDP.

Table 2 presents the estimates for the model parameters. The indices referring to recession periods are given by 0; for periods of expansion, these are characterised by 1. In addition, Σ refers to the standard error of the model.

Parameters				
	$y_t(3m)$	$y_t(12m)$	$y_t(60m)$	$\Delta_{12\%}$
$\Phi^1_{st=1}$	-0.4511	17.843	91.303	-0.0580
$\Phi^1_{st=0}$	-0.5182	64.874	140.293	-0.0007
$\Phi^2_{1,st=1}$	0.5796	-0.3082	-0.5261	0.0007
$\Phi^2_{1,st=0}$	0.7826	-0.2195	-0.4965	-0.0009
$\Phi^2_{2,st=1}$	0.6191	15.076	13.644	-0.0018
$\Phi^2_{2,st=0}$	0.2700	10.744	10.229	-0.0007
$\Phi^2_{3,st=1}$	-0.2235	-0.3028	-0.2386	0.0010
$\Phi^2_{3,st=0}$	-0.1014	-0.3272	-0.3010	0.0004
$\Phi^2_{4,st=1}$	11.459	-51.637	-55.234	-0.3883
$\Phi^2_{4,st=0}$	-20.055	0.5853	15.987	-0.0343
$\Lambda_{1,st=1}$	71.761	91.402	82.996	17.819
$\Lambda_{1,st=0}$	-0.4102	-18.327	-46.049	0.0001
$\Lambda_{2,st=1}$	-0.0585	-0.6795	-35.131	0.0175
$\Lambda_{2,st=0}$	100.437	213.419	198.502	12.502
Σ_1	0.1728	0.3780	0.3665	0.0087
Σ_0	0.8429	15.906	15.648	0.0116

Table 2: Parameters of the Model MS-VAR (1)

Observing Table 2 with the results of the estimation, we may see the differences between the two regimes. Comparing the variances of the variables, we may note that regime 1 has a lower variability of series than regime 0. In this way, Regime 1, for the expansion of the cycle, represents periods of low volatility, contrary to what occurs for Regime 0.

Figure 6 below portrays the filtered and smoothed probabilities for MS-VAR.

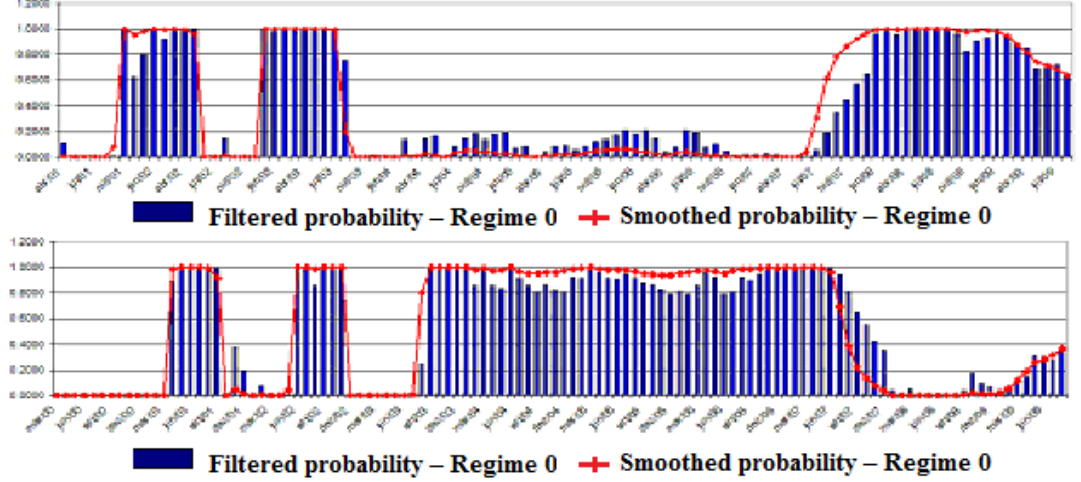


Figure 6: Filtered and Smoothed Probabilities for Brazilian Business Cycles

The MSIAH -VAR estimated above represents the following transition matrix:

$$T = \begin{bmatrix} 0.9836 & 0.0164 \\ 0.0341 & 0.9659 \end{bmatrix}$$

We thus perceive that estimated regimes are very persistent, i.e. once an economy is in a given regime, the probability of remaining within this regime is very high. In this way, the probability that the Brazilian economy is in a period of recession and will continue under the same regime for the following period is 98.36%; the probability that the Brazilian economy will be in a regime of cyclical expansion and will remain in it during the following period is 96.59%. The probability of shifting from a recessionary regime to an expansion regime is 3.41% and the contrary probability is 1.64%

Pursuant to the above probabilities, we may derive a time classification of the regimes, presented in the following table. The duration of regime 1, an expansion regime, is approximately 17 months, while the duration of 0, of recession or contraction, is approximately 5 months.

For the interpretation of the graph, a probability of 50% or lower indicates an expansion phase, while a probability exceeding 50% indicates a phase of recession or stagnation.

It may be seen in the graph with the filtered and smoothed probabilities that the expansion regime ($s_t = 1$) holds for most of the analysed period, but that there are peaks which point to periods of recession, albeit while indicating distinct periods, although we may observe the repetition of these peaks, forming areas in which the probability of being in a period of stagnation or recession is greater.

In accordance with the transition matrix, given that the economy is expanding or in a regime of recession, the probability that it remains under this regime is greater than of a change to the other regime. This indicates a lower flexibility of transition between regimes, validating the slope of the yield curve as a relevant variable for economic cycles.

Pursuant to the presented result, since 2000, Brazil has shown relatively stable economic growth. There are no significant divergences between the results of the model and the dating of the economic cycles drawn up by CODACE. This study nevertheless uses data with a monthly frequency, while the datings are drawn up on the basis of quarterly GDP. On average, the model pinpoints the peaks and troughs during the analysed period, but extends the last recessionary peak. It should be noted that the probabilities of a recession estimated by the model remained above 50% between 2008 and 2009. This does not invalidate the use of the slope component in the model. In fact, this is an expected fact, insofar as the Kalman filter uses all of the available information for generating an optimal estimate for the factor.

6 Conclusion

We propose a model which captures information from the Brazilian yield curve for the evaluation of economic cycles: A multivariate model with 2 factors, slope and proxy for economic performance, which follows two Markov processes, each one representing the faces of the securities and asset markets. The results permit the direct analysis of the relationship between the cycle phases of these two sectors. The model is used for forecasting the start and end of recessions and expansions of Brazilian GDP with a monthly frequency. The results show a strong correlation between the economy and the securities market.

The adjustment showed itself to be close to the value expected from the datings by CODACE. In summary, the components of the yield curve, estimated by Kalman filter, especially the slope factor, presents the information necessary for forecasting recessions and expansions of GDP growth.

Hence, the use of the factor estimated in a single step by Kalman filter permits improvements in forecasting economic cycles in Brazil.

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8 Annexes

Table 2: Swap DI Pré - Descriptive Statistics

	$y\mathbf{f}(3m)$	$y\mathbf{f}(6m)$	$y\mathbf{f}(9m)$	$y\mathbf{f}(12m)$	$y\mathbf{f}(15m)$	$y\mathbf{f}(18m)$	$y\mathbf{f}(21m)$	$y\mathbf{f}(24m)$	$y\mathbf{f}(27m)$	$y\mathbf{f}(30m)$	$y\mathbf{f}(33m)$	$y\mathbf{f}(36m)$	$y\mathbf{f}(39m)$	$y\mathbf{f}(48m)$	$y\mathbf{f}(60m)$
Mean	0.1535	0.1556	0.1573	0.1588	0.1607	0.1622	0.1636	0.1648	0.1662	0.1674	0.1684	0.1694	0.1703	0.1727	0.1752
Median	0.1521	0.1558	0.1559	0.1569	0.1584	0.1592	0.1588	0.1576	0.1579	0.1572	0.1567	0.1588	0.1566	0.1555	0.1572
Maximum	0.2429	0.2484	0.2574	0.2693	0.2808	0.2911	0.3011	0.308	0.315	0.3218	0.3275	0.3275	0.3375	0.3512	0.3655
Minimum	0.0825	0.0834	0.0852	0.088	0.0907	0.0922	0.0942	0.0965	0.0989	0.0986	0.0981	-12.111	0.0977	0.0972	0.0967
Standard Deviation	0.037	0.0396	0.0417	0.0436	0.0457	0.0475	0.0492	0.0507	0.0525	0.0541	0.0555	1.544	0.0580	0.0612	0.0645
Asymetry	0.2942	0.3531	0.4748	0.6118	0.7398	0.8512	0.9454	1.0196	1.0715	1.1146	1.147	-0.61	1.2008	1.2546	1.2813
Kurtosis	2.6747	2.597	2.724	2.9239	3.0902	3.2569	3.4118	3.5305	3.5819	3.6388	3.68	8.67	3.7651	3.8636	3.9024
Jarque Bera	2.1467	3.1408	4.6445	7.1382	10.4385	14.0783	17.789	21.0904	23.4246	25.5418	27.1916	3646.27	30.1774	33.4473	35.0611
Prob	[0.3419]	[0.208]	[0.0981]	[0.0282]	[0.0054]	[0.0009]	[0.0001]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.000]	[0.000]	[0.000]	[0.000]
Observations.	111	111	111	111	111	111	111	111	111	111	111	111	111	111	111

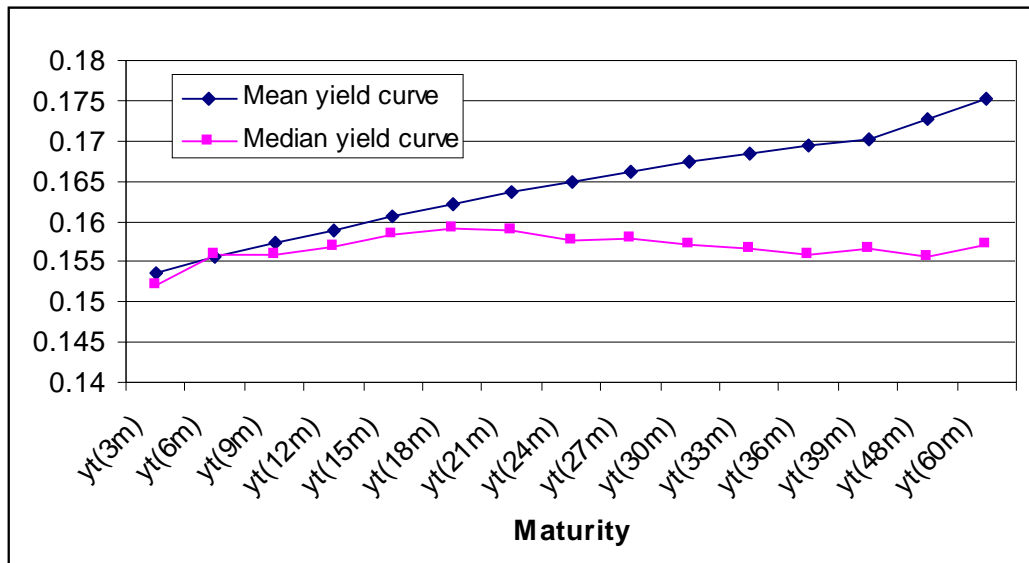


Figure 7: Brazilian Mean and Medina Yield Curves